Association Ambiguity Management in Mixed Data Dimension Tracking Problems

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ABSTRACT

Association and fusion of passive direction finding (DF) reports with active radar tracks from airborne targets is challenging because of the low dimensionality of the common kinematic measurement space. Often, multi-target scenarios lead to significant data association ambiguity. Classically, the approach to this problem is a simple hypothesis test wherein a batch of DF sensor measurements is associated with either zero or one of the radar tracks; assignment of multiple DF tracks to a single radar track is allowed without regard to compatibility, and this can lead to detrimental results. This paper develops a new approach for managing the ambiguity. The problem is formulated as a two-dimensional assignment, and any association ambiguity is determined from the k best solutions. Firm association decisions are made only when the ambiguity is at an acceptable level. The ambiguity information is also available in real time as an output to the system operator. An improved batch association score, relative to previous works, is formulated that addresses statistical correlations between individual measurement-to-track residuals; this new score is a likelihood ratio generated from Kalman Filter residuals. Where previous scoring methods lead to incorrect ambiguity assessments in certain scenarios, the new approach yields accurate results. Because the score is recursive, the batch may be extended over an arbitrary number of measurements, helping to manage association ambiguities over time. Simulation results are shown to demonstrate the algorithm.

Keywords: radar, electronic warfare, fusion, ambiguity, angle-only, multi-hypothesis, direction finding, tracking

1. INTRODUCTION

Modern intelligence, surveillance, and reconnaissance (ISR) systems involve a networked collection of sensors on multiple platforms. The capabilities of these systems are enhanced through geographic diversity of the sensor platforms and the use of multiple sensor modalities; however, the effectiveness of such systems depends on their ability to automatically fuse the multi-sensor data into a common operational picture. The specific problem addressed in this paper is the fusion of passive direction-finding (DF) reports with active radar tracks in a real-time ISR system. The association of DF data is especially challenging due to the low dimensionality of the measurements and because association ambiguities can persist over a significant amount of time. This paper develops a new approach for managing the ambiguity in the association problem.

Measurement-to-track association plays an important part in network-centric tracking systems and as a result has been a topic of much research.[1,2] Trunk and Wilson posed the DF-to-radar association problem as a multiple-hypothesis test.[3] The algorithm assumes that the DF measurements have been associated together into measurement "batches" that presume a single target. An association score is then calculated for each possible DF-batch-to-radar-track pair. The score is based on the normalized residual errors between the angle-only measurements and the predicted radar track states. To handle different numbers of DF measurements in the batches, the sum of residuals is converted into a cumulative probability of obtaining at least the observed value over N measurements, assuming a χ^2_N distribution (in this paper, we show that this assumption is not always valid). The algorithm then makes an association decision based on the probability values. The decision rules consist of three different probability thresholds and a required margin between the two best probabilities. Together, these rules form five distinct decision regions—firm correlation, tentative correlation, tentative correlation with some track (but cannot determine which), tentatively uncorrelated, and firmly uncorrelated. The thresholds were determined empirically via simulation to achieve desired error probabilities. Each DF track was considered independently with the result that more than one DF track could be associated with any given radar track.

A significant drawback of this approach is that the decision thresholds depend on the target geometry. Trunk and Wilson calculated the decision rules for a variety of two-target scenarios with different inter-target spacing; unfortunately, their results do not generalize to all possible scenarios. As a result, others have adapted this basic approach by changing the decision rules and thresholds based on different experimental techniques.[4,5] Saha mapped Trunk and Wilson's statistic into a figure of merit (FOM) rather than a cumulative probability. This new discriminant was used to analytically evaluate the probabilities of correct and false association for a given scenario[6] (Saha's score was based on the filtered DF track states rather than the DF measurements). Wang, et al., introduced a new discriminant based on a fuzzy synthetic function and derived a triple-threshold decision policy based on the statistics of the discriminant.[7]

These previous works are variations of the same basic approach, and they share the following characteristics: (i) an association score based on measurement-to-track or track-to-track residuals, (ii) a set of analytic or empirical decision rules used to detect ambiguity, and (iii) independent association decisions for each of the DF tracks. In this paper, we introduce a new algorithm that improves on the basic approach in each of these areas.

First, we note that when filtered or predicted track states—either DF or radar—are used to compute a residual error, subsequent residuals will be statistically correlated. We will show that under certain conditions, the correlation between residuals is significant and cannot be ignored. For track-to-track fusion, the correlation is due to common process noise in the tracking filters. For this problem, Saha derived an expression for the steady-state cross-correlation matrix and showed conditions where the performance of the fusion algorithm can be improved.[8] For measurement-to-track fusion, the correlated residuals are due to the correlation between track state estimates over time. In this paper, we propose a new likelihood ratio score for measurement-to-track fusion based on Kalman filter updates. The filter updates have the effect of whitening the residual errors. We show how this new likelihood ratio score leads to significantly better ambiguity estimates than the old approach.

The second problem deals with the sensitivity of the decision rules to the target geometry. Empirical methods for calculating the decision rules are impractical in real time, so one must hope that the observed geometry resembles some pre-computed scenario. Analytic methods can be applied; however, the decision rules are still difficult to compute in complex multi-target scenarios. Rather than mapping the discriminant function into difficult-to-compute decision regions, we instead pose the DF-to-radar fusion problem as a two-dimensional assignment. Given this problem formulation, a number of techniques exist for measuring the ambiguity in the solution space.[9–11] We use one such technique, based on the k best solutions, to compute the probability that a particular assignment is correct. We make a firm association decision when the probability of correct association exceeds a threshold.

Third, the assignment problem formulation allows us to consider the DF-to-radar associations jointly rather than independently. It is possible that more than one DF track originates from a single target; however, assigning the DF tracks independently means that there is no way to deal with *conflict* between assignments. For example, we may know that two different DF tracks correspond to radars that do not exist on the same target platform. The assignment problem approach lets us allow certain hypotheses but exclude others from consideration.

Our contribution is a new DF-to-radar fusion algorithm. We form a two-dimensional assignment problem using a new likelihood ration score. The solution to the assignment problem provides the best possible matching between the DF tracks and radar tracks; however, we use the k best solutions to calculate the probability that an assignment is correct. We then compare that probability to a firm decision threshold.

The remainder of this paper is organized as follows. First, we provide an overview of the improved DF-to-radar fusion algorithm. We then derive a likelihood ratio score based on residual errors. Next, we show how correlated residuals affect the statistics of the discriminant function used in the baseline approach and elaborate on the conditions when this effect is significant. We then propose a new score based on Kalman Filter updates that produce a sequence of independent residual errors. Next, we describe the k-best algorithm for measuring the ambiguity in the assignment problem. We then provide simulation results and conclude with a brief discussion.

2. DF-TO-RADAR ASSOCIATION ALGORITHM

The DF-to-radar fusion algorithm is illustrated in Figure 1. In this simple example there are two DF tracks indicated by $\mathbf{z}_i = \left[z_i(t_{i1}), ..., z_i(t_{iN_i})\right]^{\mathrm{T}}$, $i \in (1,2)$, where each track is a collection of N_i bearing measurements sampled at times $\mathbf{t}_i$. Three radar tracks are indicated by $\mathbf{X}_j = \left[\mathbf{x}_j(t_{j1}), ..., \mathbf{x}_j(t_{jM_j})\right]^{\mathrm{T}}$, $j \in (1,2,3)$, which represents a sequence of M_j

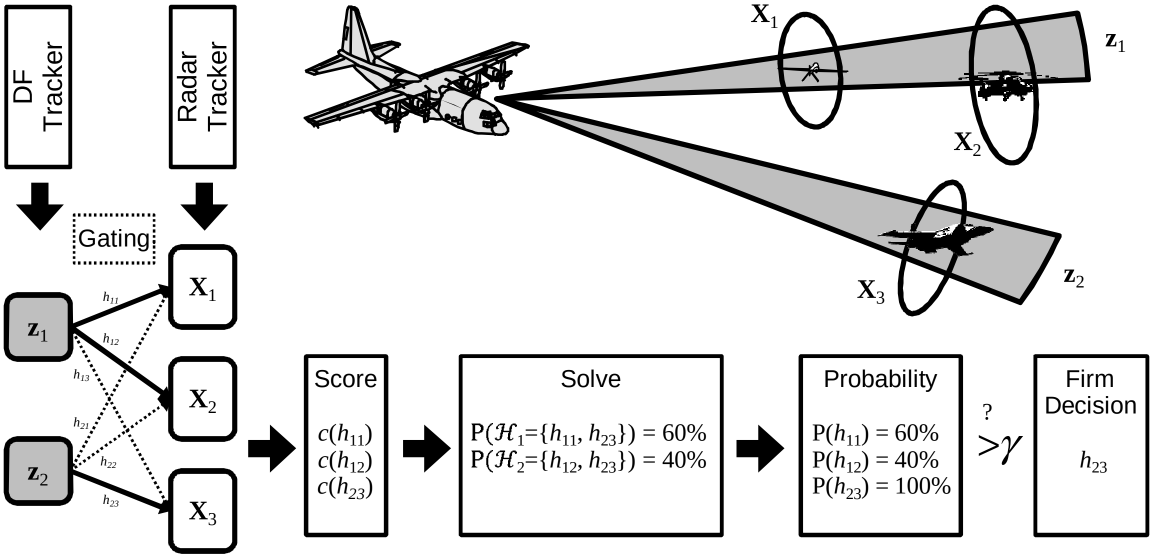

Figure 1. A block diagram of the DF-to-radar fusion algorithm, illustrating a simple example.

multi-dimensional radar measurements at times $\mathbf{t}_j$. Each of the DF measurement batches, $\mathbf{z}_i$, and radar measurement batches, $\mathbf{X}_j$, are formed by the respective DF and radar trackers and are presumed to belong to a single target. We define the association hypotheses, h_{ij}, to represent the assumption that DF track $\mathbf{z}_i$ and radar track $\mathbf{X}_j$ are the same target. In the example, there are six different association hypotheses. The cost, $c(h_{ij})$, of each of these association hypotheses is found by computing a negative log likelihood ratio (NLLR). At this point, we gate out (remove from further consideration) any h_{ij} whose cost is greater than zero; in our example, only three possibilities remain. We then solve the assignment problem and find two hypotheses, $\mathcal{H}_k$, $k \in (1,2)$, which are defined to be sets of h_{ij} that assign every DF track to zero or one radar track (in the figure, we have left off the "zero" assignments for clarity). We will show how the probability of each $\mathcal{H}_k$ is derived from its total cost, $c(\mathcal{H}_k) = \sum_{i,j:h_{ij}\in\mathcal{H}_k} c(h_{ij})$. We can then calculate the probability of each h_{ij} by summing the probabilities of the $\mathcal{H}_k$ in which it appears, $P(h_{ij}) = \sum_{k:h_{ij}\in\mathcal{H}_k} P(\mathcal{H}_k)$. For example, the association hypothesis h_{23} appears in both $\mathcal{H}_1$ and $\mathcal{H}_2$, so it has a probability of 100%; h_{11} and h_{12} each appear in one hypothesis and have probabilities of 60% and 40%, respectively. Finally, we compare $P(h_{ij})$ to a firm decision threshold, γ. In this example, only $P(h_{23})$ exceeds the threshold because the other two associations are ambiguous. Note that this approach allows firm association decisions between some pairs of tracks even if ambiguity exists elsewhere in the scenario.

For reasons of computational efficiency, gating is often done with one or more simple tests before the cost is calculated. For example, one could simply compare the angle between the DF and radar track states and reject differences that fall outside a fixed gate size. The cost acts as the final gate.

There is a tradeoff with respect to the size of the DF measurement batch. A large batch size includes more information from the track history that may be needed to resolve ambiguities, but a short batch size allows the algorithm to react quickly to certain events such as a single-sensor track swap. The use of the dimensionless likelihood ratio allows us to have the best of both worlds. Because the likelihood ratio is dimensionless, comparing scores is valid even when the batch sizes are different (in previous work,[3,6] different sized batches were handled by converting the score to a cumulative probability). As we will soon see, the likelihood ratio is calculated recursively, meaning that it is simply updated with each new measurement. This means that there is no inherent computational cost associated with a large batch. We take advantage of this as follows. With each new DF measurement, we update the likelihood score that was stored at the last iteration. We also calculate a new score based on only the last N_i measurements. The better of the two scores is used in the association algorithm and saved in memory for the next iteration. In this way, the batch size can be arbitrarily large, but it is "reset" as the most recent history may demand.

3. LIKELIHOOD RATIO DERIVATION

In the previous section, we defined the hypothesis $\mathcal{H}_k$ to be the set of DF-to-radar association hypotheses, h_{ij}, such that each DF track is assigned to zero or one radar track. Even in a simple scenario, we must consider many different hypotheses. For example, if both the DF sensor and radar detect two targets, then there are eight possible associations leading to the seven different hypotheses shown in Table 1 (assuming single assignment). The association event h_{i0} indicates that DF track i does not associate with any of the radar tracks, and the event h_{0j} indicates that radar track j does not associate with any of the DF tracks.

Associations		h_{10}	h_{20}	h_{01}	h_{02}	h_{11}	h_{12}	h_{21}	h_{22}
Hypotheses	4 Targets	✓	✓	✓	✓				
	3 Targets		✓		✓	✓			
	3 Targets		✓	✓			✓		
	3 Targets	✓			✓			✓	
	3 Targets	✓		✓					✓
	2 Targets					✓			✓
	2 Targets						✓	✓	

Table 1. Seven Hypotheses for the Two-Sensor, Two-Target Scenario

In order to form an assignment problem, the cost of each $\mathcal{H}_k$ must be the sum of the costs of the individual association hypotheses,[12,13]

$$c(\mathcal{H}_k) = \sum_{i,j:h_{ij}\in\mathcal{H}_k} c(h_{ij}). \tag{1}$$

One way to form such a cost is to use the negative log likelihood ratio, $c(h_{ij}) = NLLR(h_{ij})$. We derive the NLLR by first defining the likelihood, $\mathcal{L}$, of the following association hypotheses:

$$\mathcal{L}(h_{ij}) = P_N \cdot P_D^{(\text{DF})}(h_{ij}) \cdot P_D^{(\text{radar})}(h_{ij}) \cdot f(\mathbf{z}_i, \mathbf{X}_j | h_{ij}) \tag{2}$$

$$\mathcal{L}(h_{i0}) = P_N \cdot P_D^{(\text{DF})}(h_{i0}) \cdot \left(1 - P_D^{(\text{radar})}(h_{i0})\right) \cdot f(\mathbf{z}_i | h_{i0}) \tag{3}$$

$$\mathcal{L}(h_{0j}) = P_N \cdot \left(1 - P_D^{(\text{DF})}(h_{0j})\right) \cdot P_D^{(\text{radar})}(h_{0j}) \cdot f(\mathbf{X}_j | h_{0j}) \tag{4}$$

In these expressions, P_N is the probability of a new target, $P_D(h_{ij})$ is the probability that the DF sensor or radar (as indicated) detected the target presumed in h_{ij}, and f is the probability density function of the measurements given the presumed target (explained later). It is important to emphasize that the probability of detection, $P_D(h_{ij})$, for both the DF and radar sensors, depends on the presumed target. For example, the radar cannot detect certain DF targets that are beyond the radar's detection range (due to the inherent range advantage of passive DF). On the other hand, the DF sensor cannot detect a radar target unless the target's radar is actively transmitting. Often, the information needed to calculate these values is not known, so we make certain assumptions. If we assume that $P_D(h_{ij}) = P_D$ for a particular sensor, then these terms will cancel out in the likelihood ratio and can be ignored.

With these definitions, we can compute the likelihood of any hypothesis,

$$\mathcal{L}(\mathcal{H}_k) = \prod_{i,j:h_{ij}\in\mathcal{H}_k} \mathcal{L}(h_{ij}). \tag{5}$$

We can separate the individual association hypotheses in $\mathcal{H}_k$ into three distinct groups representing joint DF and radar detections, DF-only detections, and radar-only detections. The likelihood is then written

$$\mathcal{L}(\mathcal{H}_k) = \prod_{i>0,j>0:h_{ij}\in\mathcal{H}_k}^{N_k} \mathcal{L}(h_{ij}) \prod_{i:h_{i0}\in\mathcal{H}_k}^{N_{\text{DF}}-N_k} \mathcal{L}(h_{i0}) \prod_{j:h_{0j}\in\mathcal{H}_k}^{N_{\text{radar}}-N_k} \mathcal{L}(h_{0j}), \tag{6}$$

where we have added to our notation the total number of events in each group (N_k is the number of joint DF and radar detections in hypothesis k, N_{DF} is the total number of DF tracks, and N_{radar} is the total number of radar tracks).

We define the null hypothesis to be that which contains no DF-to-radar associations, or $N_k = 0$ (e.g., the four-target hypothesis in Table 1). The likelihood of the null hypothesis can be separated into two distinct groups representing the DF-only detections and radar-only detections,

$$\mathcal{L}(\mathcal{H}_0) = \prod_{i=1}^{N_{\text{DF}}} \mathcal{L}(h_{i0}) \prod_{j=1}^{N_{\text{radar}}} \mathcal{L}(h_{0j}). \tag{7}$$

The likelihood ratio is defined to be

$$LR(\mathcal{H}_k) = \frac{\mathcal{L}(\mathcal{H}_k)}{\mathcal{L}(\mathcal{H}_0)}. \tag{8}$$

Note that all of the terms in $\mathcal{L}(\mathcal{H}_k)$ that correspond to DF-only or radar-only events are also present in the null hypothesis. After canceling common terms, all that remains in the numerator is a product over N_k association hypotheses. In the denominator, we cancel all but N_k of the DF-only and radar-only hypotheses. Therefore, the likelihood ratio reduces to a product of N_k terms,

$$LR(\mathcal{H}_k) = \prod_{i>0,j>0:h_{ij}\in\mathcal{H}_k}^{N_k} \frac{P_D^{(\text{DF})}(h_{ij}) P_D^{(\text{radar})}(h_{ij}) f(\mathbf{z}_i, \mathbf{X}_j | h_{ij})}{P_N P_D^{(\text{DF})}(h_{i0})(1 - P_D^{(\text{radar})}(h_{i0})) P_D^{(\text{radar})}(h_{0j})(1 - P_D^{(\text{DF})}(h_{0j})) f(\mathbf{z}_i | h_{i0}) f(\mathbf{X}_j | h_{0j})}. \tag{9}$$

We define the likelihood ratio of the individual association hypotheses as follows:

$$LR(h_{ij}) = \begin{cases} \frac{P_D^{(\text{DF})}(h_{ij}) P_D^{(\text{radar})}(h_{ij}) f(\mathbf{z}_i, \mathbf{X}_j | h_{ij})}{P_N P_D^{(\text{DF})}(h_{i0})(1 - P_D^{(\text{radar})}(h_{i0})) P_D^{(\text{radar})}(h_{0j})(1 - P_D^{(\text{DF})}(h_{0j})) f(\mathbf{z}_i | h_{i0}) f(\mathbf{X}_j | h_{0j})} & i, j \neq 0 \\ 1 & \text{otherwise} \end{cases}. \tag{10}$$

This allows us to write the likelihood ratio of the hypothesis $\mathcal{H}_k$ as a product of likelihood ratios for the individual association hypotheses,

$$LR(\mathcal{H}_k) = \prod_{i,j:h_{ij}\in\mathcal{H}_k} LR(h_{ij}). \tag{11}$$

Finally, taking the negative log of the likelihood ratio satisfies (1), allowing the formulation of a two-dimensional assignment problem.

We now return to the measurement density function, $f(\mathbf{z}_i, \mathbf{X}_j | h_{ij})$, where again h_{ij} indicates the hypothesis that the measurements $\mathbf{z}_i$ and $\mathbf{X}_j$ belong to the same target. In previous works, the model for h_{ij} was simply the filtered radar track state. We will represent the filtered radar track state by $\hat{\mathbf{x}}_j(t)$ and note that the tracking space of $\hat{\mathbf{x}}_j(t)$ is usually different than the measurement space of $\mathbf{x}_j(t)$. With this model, $f(\mathbf{z}_i, \mathbf{X}_j | h_{ij})$ becomes $f(\mathbf{z}_i | \hat{\mathbf{X}}_{ij}) f(\mathbf{X}_j | h_{ij})$, where the rows of $\hat{\mathbf{X}}_{ij}$ are the track state estimates propagated to the bearing measurement times $\mathbf{t}_i$, and $f(\mathbf{X}_j | h_{ij})$ is independent of $f(\mathbf{z}_i | h_{ij})$. Since $f(\mathbf{X}_j | h_{ij})$ appears in both the numerator and denominator of the likelihood ratio, it cancels out.

For notational convenience, we define the cross-covariance of track state estimates $\hat{\mathbf{x}}_j(t_r)$ and $\hat{\mathbf{x}}_j(t_c)$, which are both rows of $\hat{\mathbf{X}}_{ij}$, to be $\mathbf{P}_{ij}(t_r, t_c)$. In general, the calculation of these terms requires careful bookkeeping. For example, consider the covariance between two track state estimates at update times t_1 and t_2. Just before the update, at time t_2^-, the track state estimates are related by a simple forward projection; their covariance is

$$\mathbf{P}(t_2^-, t_1) = \mathbf{\Phi} \mathbf{P}(t_1, t_1), \tag{12}$$

where $\mathbf{\Phi}$ represents the dynamic model from time t_1 to t_2. After the update, at time t_2, the estimates have covariance

$$\mathbf{P}(t_2, t_1) = (\mathbf{I} - \mathbf{K}\mathbf{H}(t_2)) \mathbf{\Phi} \mathbf{P}(t_1, t_1), \tag{13}$$

where $\mathbf{K}$ is the Kalman gain and $\mathbf{H}(t)$ is the Jacobian of the measurement function at time t. These terms become increasingly complicated with more projections and updates. In the next section, we show certain circumstances where the cross-covariance terms have little effect and can be ignored; in Section 5, we present an alternative algorithm that avoids these difficult calculations altogether.

Assuming a normal distribution, the density function $f(\mathbf{z}_i|\hat{\mathbf{X}}_{ij}) \sim \mathcal{N}(\hat{\mathbf{z}}_{ij}, \Sigma_{ij})$ is evaluated by estimating the bearing measurements from the radar track state, $\hat{\mathbf{z}}_{ij} = h\left(\hat{\mathbf{X}}_{ij}\right)$. The elements of $\text{cov}(\hat{\mathbf{z}}_{ij})$ are approximated by

$$[\text{cov}(\hat{\mathbf{z}}_{ij})]_{r,c} = \mathbf{h}(t_r)\mathbf{P}_{ij}(t_r, t_c)\mathbf{h}^{\mathrm{T}}(t_c), \tag{14}$$

where $\mathbf{h}(t)$ is the Jacobian of the one-dimensional bearing measurement function, h, at time t. The covariance $\mathbf{\Sigma}_{ij} = \text{cov}(\hat{\mathbf{z}}_{ij}) + \mathbf{R}_i$. combines the covariance of $\hat{\mathbf{z}}_{ij}$ with the bearing measurement covariance, $\mathbf{R}_i$. The Mahalanobis (statistical) distance from the DF measurement sequence to the estimated bearings is

$$d_{ij} = \sqrt{\left(\mathbf{z}_i - \hat{\mathbf{z}}_{ij}\right)^{\mathrm{T}} \mathbf{\Sigma}_{ij}^{-1} \left(\mathbf{z}_i - \hat{\mathbf{z}}_{ij}\right)}. \tag{15}$$

The measurement density is

$$f(\mathbf{z}_i|\hat{\mathbf{X}}_{ij}) = \frac{e^{-d_{ij}^2/2}}{\sqrt{|2\pi\mathbf{\Sigma}_{ij}|}}. \tag{16}$$

In previous works,[3–7] it was assumed that the track state estimates at different times were independent (i.e. $\mathbf{P}_{ij}(t_r, t_c) = 0$ for $r \neq c$). If this assumption is true, then the covariance matrix Σ_{ij} is diagonal, and the density function simplifies to

$$f(\mathbf{z}_i|\hat{\mathbf{X}}_{ij}) = \prod_{t \in \mathbf{t}_i} \frac{e^{-(z_i(t) - \hat{z}_{ij}(t))^2/2\sigma_{ij}^2(t)}}{\sqrt{2\pi}\sigma_{ij}(t)}, \tag{17}$$

where $\sigma_{ij}(t)$ is the diagonal element of Σ_{ij} associated with bearing measurement $z_i(t)$.

It is important to emphasize that the densities in (16) and (17) correspond to fusing a single DF measurement batch with a single radar track, assuming h_{ij}; the calculation consists of N_i residual errors, one for each of the bearing measurements in $\mathbf{z}_i$. The residual errors are *correlated* because the filtered, predicted track state estimates, $\hat{\mathbf{x}}_j(t)$, are correlated in time.

4. CORRELATED RESIDUALS

The residual errors in (17) may be considered independent if the off-diagonal terms of Σ_{ij} are sufficiently small. The critical parameters turn out to be the relative measurement update rate and relative measurement accuracy between the radar and DF sensor. For example, if we receive multiple DF measurements for every radar update, the propagated radar track states are based on the same measurement information and are highly correlated. Even then, the correlation terms only matter if the DF measurements are relatively accurate. For inaccurate DF measurements, the DF measurement covariance, $\mathbf{R}$, loads the diagonal of Σ to the point where the off-diagonal terms become insignificant.

A simulation was used to explore the parameter space between relative measurement rate and relative measurement accuracy. The results indicate when one is justified in ignoring the correlation terms in the likelihood function—in other words, when one may use (17) instead of (16). The simulation used a bearing-only sensor and a range/bearing radar. The initial range to the target was fixed, but the initial target heading was varied over a number of Monte Carlo trials. An extended Kalman Filter (EKF) with a nearly constant velocity (NCV) motion model was used to track the target using only the radar measurements. The residual of each bearing measurement was calculated based on the predicted radar track state; the bearing measurements were then grouped into batches of ten measurements, and the batch score was calculated to be the sum of the individual measurement residuals.

A set of parameters was chosen to illustrate the effect of the correlated radar track states. The DF sensor updates once per second, the radar updates every ten seconds, and both sensors have a bearing measurement accuracy of one degree. Figure 2 shows the cumulative probability of three different statistics. First, we plot the Kalman Filter innovation for the radar tracker. Since the radar measurements are two-dimensional, we expect this statistic to follow a χ^2 distribution with two degrees of freedom (a χ^2 distribution with N degrees of freedom is denoted χ_N^2). The statistic for the individual bearing measurements is the Mahalanobis (statistical) distance between the bearing measurement and the radar track state at that time. Since these measurements are one-dimensional, we would expect this statistic to follow a χ_1^2 distribution if the residuals were independent. The batch statistic is the sum of ten individual bearing measurement residuals. Again assuming independence, we would expect this statistic to follow a χ_{10}^2 distribution.

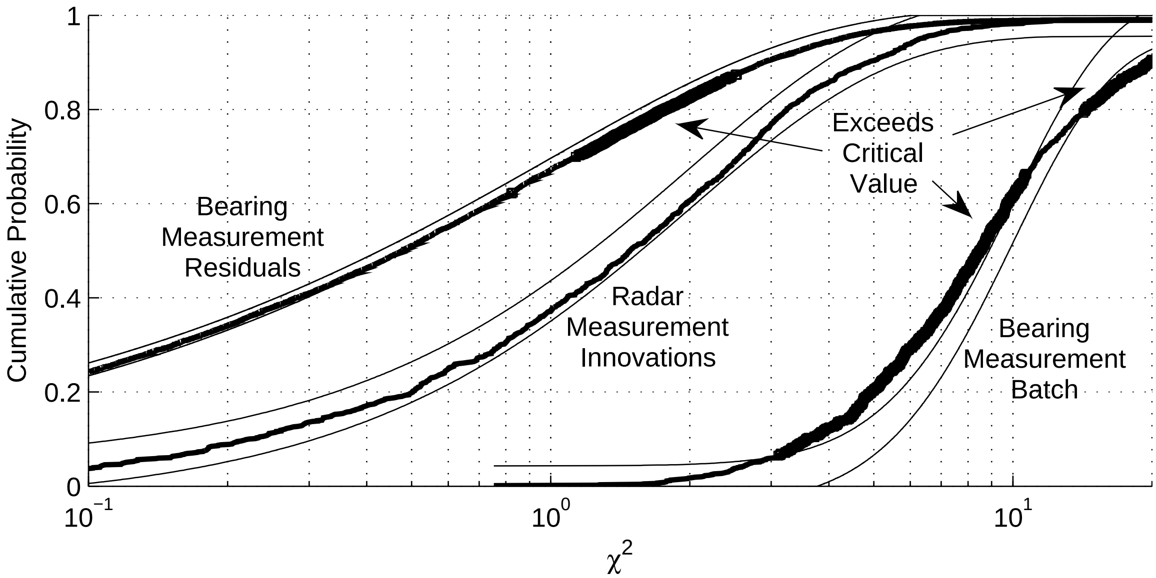

Figure 2. This figure shows the cumulative probability of various statistics. The radar measurement residual errors follows a χ_2^2 distribution as expected. If we assume independence, we expect the individual bearing residuals to follow a χ_1^2 distribution and the batch statistic, which is the sum of ten individual bearing residuals, to follow χ_{10}^2; however, both of these distributions fall outside the expected bounds where marked, indicating that we should reject the independent residual hypothesis.

The expected distributions (χ_1^2, χ_2^2, and χ_{10}^2) are indicated by a boundary at the critical value for a Kolmogorov-Smirnov (KS) goodness-of-fit test (at the 5% significance level). The KS test is used to determine whether the actual data could have been generated by the expected probability distribution (the null hypothesis). The test checks the maximum distance between the null hypothesis and the cumulative distribution of the data against a critical value. Data generated from the null hypothesis is expected to exceed the critical value less than some small percentage of the time (typically 5%), so one may reject the null hypothesis if the critical value is exceeded.

We see that the distribution of radar measurement innovations falls within the respective critical values, so we do not reject the hypothesis that they are independent—exactly what one expects from a Kalman filter. On the other hand, both the individual bearing measurement residuals and the batch statistic exceeds the critical value where indicated, so we reject the hypothesis of independent residual errors—in other words, the correlation terms are statistically significant and should not be ignored.

We now explore the parameter space to see where the batch statistic exceeds the critical value by running a large number of Monte Carlo trials. The radar update time (ten seconds) and accuracy (one degree) are held constant while the corresponding DF parameters were varied. Figure 3 roughly separates this parameter space into two regions: above the line, the residuals can be considered independent; below the line, correlation terms are statistically significant. These results confirm our previous claim that correlated residuals occur when the DF measurement error is relatively small or the DF measurement rate is relatively fast. In particular, is seems that we can consider the residuals to be independent any time the DF measurement error is over five times the radar bearing measurement error or any time the DF measurement rate is less than or equal to that of the radar. In Section 7, we will present results where the radar and DF sensor have equal bearing error, and the DF sensor has twice the measurement rate of the radar—these parameters fall squarely in the region where correlated residuals are a factor, as indicated in the figure.

5. IMPROVED SCORE BASED ON KALMAN FILTER UPDATES

We have just seen the effect when the residual covariance, $\mathbf{\Sigma}$, is not diagonal but has nonzero off-diagonal terms. Note that one might consider smoothing the track state estimate over the batch interval in an attempt to minimize the correlation between any two track state estimates. For example, consider a batch fit of the radar measurements to some dynamic model. The covariance terms then become the covariance of the model fit transformed to measurement space; the result is

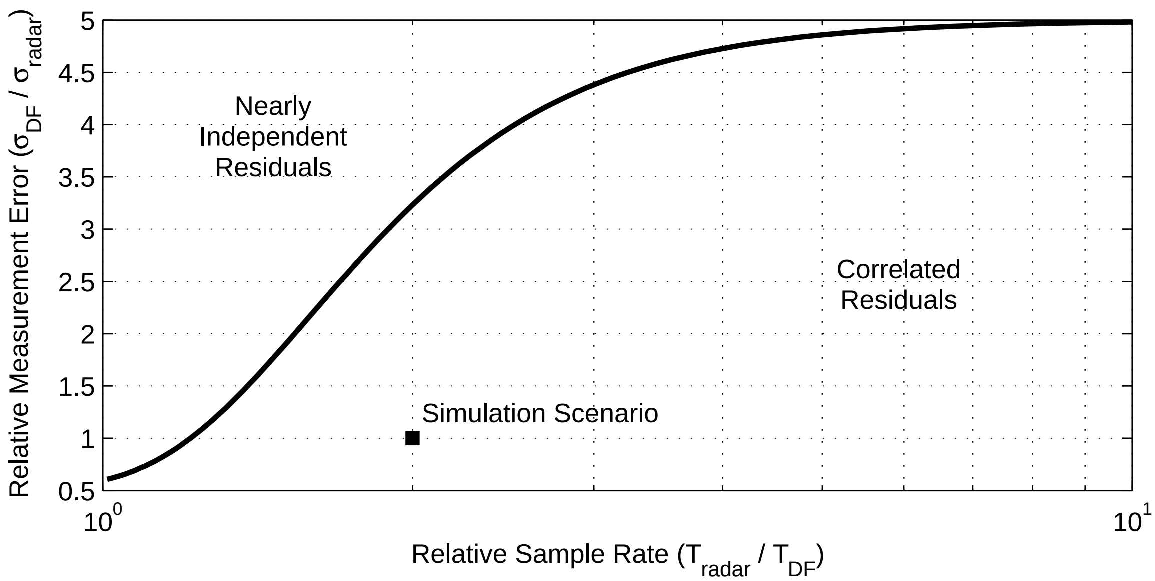

Figure 3. A rough separation of the relative angle error and relative measurement rate parameter space into regions where the residuals can be considered independent (above the line) and where the correlation terms are significant (below the line). The simulation in Section 7 uses the parameters indicated by a square.

that all of the elements of $\mathbf{\Sigma}$ will be more-or-less equal. The effect is to spread out the correlation over time, but it does not reduce its impact. We conclude that it doesn't particularly matter which smoothing technique is used, if any—what matters is that the statistics of the residuals are properly addressed.

When we are not justified in simply ignoring the correlation terms but want to avoid the cross-covariance calculations in (16), we need to use a different model for h_{ij}. Instead of forming a track with just the radar measurements, consider forming a track with both the DF and radar measurements. When we update this track state with either a DF or radar measurement, the innovations sequence is statistically uncorrelated. We define $\mathbf{z}(t)$ to be either a DF or radar measurement at time t, $\hat{\mathbf{x}}(t)$ to be the track state estimate just before the update, $\mathbf{P}_{ij}(t)$ to be the corresponding track state covariance, and $\Sigma_{ij}(t) = \mathbf{h}(t)\mathbf{P}_{ij}(t)\mathbf{h}^{\mathrm{T}}(t) + \mathbf{R}(t)$; $\mathbf{h}(t)$ is the Jacobian of the measurement function and $\mathbf{R}(t)$ is the measurement covariance. We also define $d_{ij}^2(t) = (\mathbf{z}(t) - h(\hat{\mathbf{x}}(t)))^{\mathrm{T}} \Sigma_{ij}(t) (\mathbf{z}(t) - h(\hat{\mathbf{x}}(t)))$. With these definitions, we can compute the measurement density

$$f(\mathbf{z}_i, \mathbf{X}_j | h_{ij}) = \prod_{t \in (\mathbf{t}_i, \mathbf{t}_j)} \frac{e^{-d_{ij}^2(t)/2}}{\sqrt{|2\pi\Sigma_{ij}(t)|}}. \tag{18}$$

Because this result is recursive, it leads to an efficient implementation. In the algorithm, we initiate a track for each association hypothesis h_{ij}. Then as new DF and radar measurements arrive, we update both the track state and the cumulative track score. These scores are used to form the assignment problem costs, and the assignment solutions are used to determine the unambiguous association hypotheses. Once we have made a firm association decision, we simply forward the corresponding tracks to the system operator. The remaining tentatively associated tracks can also be made available to the operator with their association probabilities.

The following is a proof of the independence of the residual sequence for a linear Kalman Filter. The concept is the same for the nonlinear Extended Kalman Filter (EKF) although the linearization step may make it somewhat suboptimal.

We are interested in the covariance between two residual errors at updates k and $k+1$; the errors are

$$e_k = z_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1} \tag{19}$$

$$e_{k+1} = z_{k+1} - \mathbf{H}_{k+1}\mathbf{\Phi}_{k+1}\left(\hat{\mathbf{x}}_{k|k-1} - \mathbf{K}_k e_k\right). \tag{20}$$

Since the errors have zero mean, the covariance is given by

$$E\left[e_{k+1}e_k\right] = E\left[\left(z_{k+1} - \mathbf{H}_{k+1}\mathbf{\Phi}_{k+1}\left(\hat{\mathbf{x}}_{k|k-1} - \mathbf{K}_k\left(z_k - \mathbf{H}_k\hat{\mathbf{x}}_{k|k-1}\right)\right)\right)\left(z_k - \hat{\mathbf{x}}_{k|k-1}^{\mathrm{T}}\mathbf{H}_k^{\mathrm{T}}\right)\right]. \tag{21}$$

Eliminating independent terms with zero expected value reduces the expression to

$$E\left[e_{k+1}e_k\right] = E\left[-\mathbf{H}_{k+1}\mathbf{\Phi}_{k+1}\left(\mathbf{K}_k\left(\mathbf{H}_k\hat{\mathbf{x}}_{k|k-1}\hat{\mathbf{x}}_{k|k-1}^{\mathrm{T}}\mathbf{H}_k^{\mathrm{T}} + z_k^2\right) - \hat{\mathbf{x}}_{k|k-1}\hat{\mathbf{x}}_{k|k-1}^{\mathrm{T}}\mathbf{H}_k^{\mathrm{T}}\right)\right] \tag{22}$$

$$= -\mathbf{H}_{k+1}\mathbf{\Phi}_{k+1}\left(\mathbf{K}_k\left(\mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k\right) - \mathbf{P}_{k|k-1}\mathbf{H}_k^{\mathrm{T}}\right). \tag{23}$$

If we substitute the Kalman gain,

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k\left(\mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k\right)^{-1}, \tag{24}$$

into (23), we see that the covariance goes to zero.

6. ASSOCIATION AMBIGUITY

The association of DF measurements to radar tracks is done by solving a two-dimensional assignment problem. Murty[14,15] previously introduced an algorithm that finds multiple quality solutions to the 2D assignment problem where each alternative solution represents a different complete data association hypothesis $\mathcal{H}_k$. In the following, let $\{\mathcal{H}_1, \mathcal{H}_2, \ldots, \mathcal{H}_K\}$ denote a set of K *ranked* solutions to the assignment problem such that $\mathcal{H}_1$ corresponds to the "best" solution returned from the assignment solver, $\mathcal{H}_2$ to the "second-best", etc. (this approach is commonly called "k-best"). Ranking is done by cost,

$$c(\mathcal{H}_1) \leq c(\mathcal{H}_2) \leq \ldots \leq c(\mathcal{H}_K), \tag{25}$$

where the costs, $c(\mathcal{H}_k)$, are defined as negative log-likelihood ratios. As long as K is sufficiently large, it is the case that

$$\sum_{k=0}^{K} P(\mathcal{H}_k) \approx 1, \tag{26}$$

and therefore

$$P(\mathcal{H}_k) \approx \frac{P(\mathcal{H}_k)}{\sum_{k=0}^{K} P(\mathcal{H}_k)}. \tag{27}$$

We can now use (27) to obtain

$$P(\mathcal{H}_k) = \frac{e^{-c(\mathcal{H}_k)}}{1 + \sum_{k=1}^{K} e^{-c(\mathcal{H}_k)}}, \tag{28}$$

which gives the probability of a hypothesis in terms of its cost $c(\mathcal{H}_k)$.[1,16] Note that (25) implies

$$P(\mathcal{H}_1) \geq P(\mathcal{H}_2) \geq \ldots \geq P(\mathcal{H}_K). \tag{29}$$

It may be necessary to "shift" the costs in (28) for numerical stability:

$$P(\mathcal{H}_k) = \frac{e^{-c(\mathcal{H}_k)+\alpha}}{e^{\alpha} + \sum_{k}^{K} e^{-c(\mathcal{H}_k)+\alpha}}, \tag{30}$$

where a possible choice for the cost shift is $\alpha = c(\mathcal{H}_1)$.[16]

The probabilities of the individual association hypotheses, $P(h_{ij})$, can be calculated by adding the probability of all the hypotheses $\mathcal{H}_k$ that contain h_{ij},

$$P(h_{ij}) \approx \sum_{k: h_{ij} \in \mathcal{H}_k} P(\mathcal{H}_k). \tag{31}$$

The assignment probabilities $P(h_{ij})$ serve as a measure of the *association uncertainty* or *ambiguity*. If $P(h_{ij}) < 1$, then there is a non-zero probability that the association from the best hypothesis $\mathcal{H}_1$ is incorrect. In practice, we require $P(h_{ij})$ to exceed some threshold before making a firm association decision, and we hold all other association decisions in abeyance until the ambiguity is resolved.

Two types of ambiguity may occur. The first type of ambiguity is when one DF track associates with more than one radar track. This type of ambiguity occurs when several targets share a common line of bearing. In this case, we defer the association decision until the ambiguity is resolved. On the other hand, it is possible that a previously resolved scenario again becomes ambiguous. In this case, we leave the firm association decision in place unless there is also ambiguity in the DF measurement-to-track associations or the radar measurement-to-track associations. For example, as long as two radar tracks are separable in range and two DF tracks by their features, it doesn't matter if the bearing becomes ambiguous—the previous track-to-track associations, made when the bearings were unambiguous, remains valid.

The second type of ambiguity occurs when more than one DF track associates with a single radar track. For example, a DF sensor may establish independent tracks on multiple emitters from the same target (or the same emitter simultaneously operating in multiple modes), and we should associate all such tracks with a single target. This situation can be handled by allowing multiple assignment in the assignment solver and eliminating solutions with conflicting assignments. Another situation exists when two targets are at the same bearing but the radar only detects one of them. This can occur in practice because of the inherent range advantage of the passive DF sensor. If there is no conflict between the DF tracks, then there is no way to distinguish this situation from the previous (where the DF tracks belong to the same target) until the radar establishes a track on the second target. This example emphasizes the importance of continuing to monitor the ambiguity in the scenario, even after a firm decision has been made.

7. SIMULATION RESULTS

The simulation scenario has a radar and DF sensor on a single platform. There are two targets at different ranges but approximately the same bearing. The radar updates every ten seconds and the DF sensor every five seconds with the bearing measurements associated based on the target emitter's RF. Both the radar and DF sensor have one degree bearing measurement errors. According to Figure 3, this set of parameters produces correlated residuals unless the improved association score is used.

In Figure 4, the plot on the left shows the true bearings (solid lines) and bearing measurements (scattered dots), with the two targets differentiated by color. The plot on the right shows the probability of the correct association hypothesis as a function of time. The algorithm was run in two different modes: in the first mode (indicated by the set of black lines), the association score was computed recursively over the entire scenario; in the second mode (indicated by the set of gray lines), the radar track was filtered through only the last sixty seconds of DF measurements. For both modes, the jagged line indicates a single Monte Carlo trial, and the smooth line is the average over a large number of trials.

For the recursive mode, we see that on average the initial association probability is 70%, increases slightly, then remains flat as the bearings cross; when the bearings separate again, the association probability increases again until the 95% threshold is met at about 270 seconds. For the sixty second batch mode, the initial association probability is the same, 70%; however, as previous data is forgotten, the association probability now drops to 50% at the exact time that the bearings cross. Once the targets separate, the association probability increases again until it crosses the threshold at about 320 seconds. We see that the recursive mode allows us to make a firm association decision approximately fifty seconds before the batch mode.

We also ran this scenario using the previous algorithm based on DF-measurement-to-radar-track residuals without the whitening effect of the Kalman filter updates. Using this score, the measured association probabilities were either 0% or 100% with very little in between. The effect of the correlated residuals was to produce a measure of association ambiguity that did not accurately represent the true ambiguity in the scenario.

8. CONCLUSION

We present a unique algorithm for the association of DF tracks to radar tracks that addresses the ambiguity in the association decision. The association problem is formulated as a two-dimensional assignment where the potential associations are scored using the likelihood ratio. The assignment problem is solved, and the k best solutions are used to calculate the probability that a particular association is correct. We compare this probability to a firm decision threshold.

Our approach improves on the previous work in three areas. First, we described the previous approach to forming an association score based on the residuals between the DF measurements and the filtered radar track states. We showed that this score is difficult to compute unless the residuals are independent. We noted that the residuals may be considered independent in certain scenarios where the DF measurements have little impact on the radar track state because of the relative data rate or measurement accuracy. In the general case, however, this assumption can lead to inaccurate ambiguity assessments. We proposed an improved association score based on the normalized Kalman filter innovations of a track that

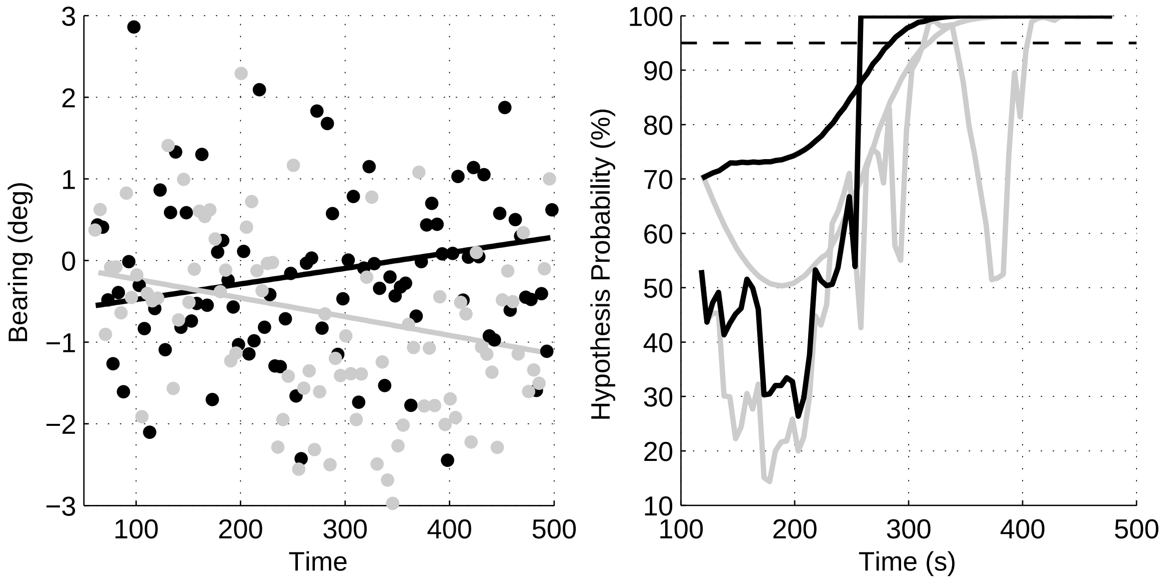

Figure 4. Simulation results for a platform with DF and radar sensor tracking two targets at nearly the same bearing. The left plot shows the true bearings (solid lines) and DF measurements (scattered dots); the two targets are differentiated by color. The right plot shows the association probability as a function of time for two different modes: (i) a recursive association score (black lines) and (ii) a sixty second batch (gray lines). Both cases show the result of a single Monte Carlo trail and the average over a large number of trials. A firm association decision is made when the probability crosses the indicated threshold (dashed).

is updated with both DF and radar measurements. This score can be computed recursively, as in the previous approach, which allows the algorithm to consider an arbitrarily large batch size. We showed that this score provides accurate results in a scenario with significant ambiguity.

Second, our approach provides a direct assessment of the ambiguity through the individual association probabilities, and our decision is based on a simple threshold. In previous approaches, the association score was mapped into different decision regions that were both difficult to compute and specific to certain scenarios. In some cases, the decision boundaries were empirically determined to yield the desired error probabilities. In this way, the desired performance was achieved despite the fact that the statistics of the discriminant function deviated from the assumed distribution. A result of this is that the decision boundaries were valid only for a particular set of parameters—a testament to the many variations of this approach found in the literature.

Third, we note that in previous approaches, each DF-to-radar association was considered independently of the others which could lead to potentially conflicting assignments. In the assignment problem approach, we solve the problem jointly which allows conflict between multiple assignments to be addressed.

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